Solution to Physics 9GA MT#2 (2020 W)

1-(a) $C_{23} = C_{20} + C_{30} = 20\,\mu F$

$$C_{1234}^{-1} = C_{10}^{-1} + C_{23}^{-1} + C_{40}^{-1}$$

$$= \frac{1}{10\,\mu F} + \frac{1}{20\,\mu F} + \frac{1}{20\,\mu F} = \frac{1}{5\,\mu F}$$

$$\therefore C_{1234} = 5\,\mu F_{\divideontimes}$$

1-(b). The charges stored on C_{10} and on C_{23} are the same as the charge stored on the network capacitor C_{1234}

$$Q_{10} = Q_{23} = Q_{1234}$$

Since $Q_{1234} = V_{ab} \cdot C_{1234}$

$$Q_{1234} = (60V) \cdot (5\,\mu F) = 300\,\mu C$$

The potential difference across C_{10} is

$$\Delta V_{10} = \frac{Q_{10}}{C_{10}} = \frac{300\,\mu C}{10\,\mu F} = 30V$$

The potential difference across C_{20} is the same as that across C_{23}:

Thus

$$\Delta V_{20} = \Delta V_{23} = \frac{Q_{23}}{C_{23}} = \frac{300\mu C}{20\mu F}$$

$$= 15\,V$$

1-(c) The energies stored on C_{20} and C_{30} are as follows

$$U_{20} = \frac{C_{20}}{2}(\Delta V_{20})^2 = \frac{1}{2}(5\mu F)\cdot(15V)^2$$

$$= 562.5\,\mu J$$

$$U_{30} = \frac{C_{30}}{2}(\Delta V_{30})^2 = \frac{C_{30}}{2}(\Delta V_{23})^2$$

$$= 3\,U_{20} = 1.69\times 10^{-3}\,J$$

1-(d) We need to know the charge Q_{1234} after the dielectric is inserted in C_{10}.

$$C_1 = K\cdot C_{10} = 20\mu F$$

$$C_{1234}^{-1} = \frac{1}{C_1} + \frac{1}{C_{23}} + \frac{1}{C_{40}} = \frac{3}{20\mu F}$$

Thus

$$C'_{1234} = \frac{20\mu F}{3} = 6.67\mu F$$

$$Q'_{1234} = V_{ag}\, C'_{1234} = (60V)\left(\frac{20\mu F}{3}\right)$$

$$= 400\mu C$$

$$\therefore \Delta V_1 = \frac{Q'_{1234}}{C_1} = \frac{400\mu C}{20\mu F} = 20V$$

1-(e) In this case, the charges on all capacitors remain unchanged.

The potential drop across C_1 is simply

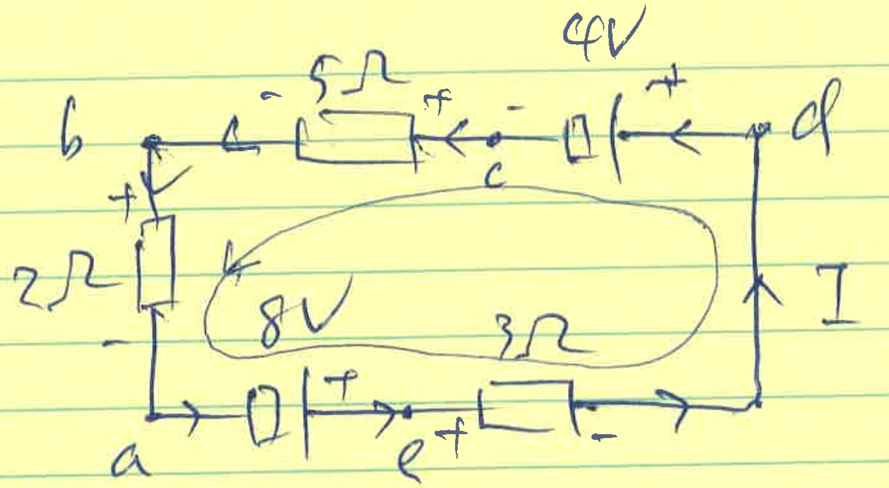
$$\Delta V_1 = \frac{Q_{1234}}{C_1} = \frac{300\mu C}{20\mu F} = 15V$$

The potential drop across C_{40} is

$$\Delta V_{40} = \frac{Q_{1234}}{C_{40}} = \frac{300\mu F}{20\mu F} = 15V$$

because the charge on C_{40} is the same as the charge on the network capacitor in (a)

2-(a)



Assign the current counter-clockwise. Choose a counter-clockwise loop as shown. Starting from a:

$$\oint_{aedcba} \vec{E}\cdot d\vec{l} = 0 = V_{ae} + V_{ed} + V_{dc} + V_{cb} + V_{ba}$$

$$\Rightarrow \; -8 + 3I + 4 + 5I + 2I = 0$$

$$\therefore \; 10I = 4$$

$$I = 0.4\,A$$

2-(b) $P_{5\Omega} = I^2(5\Omega) = (0.4A)^2(5\Omega) = 0.8$ Watt.

2-(c) $V_{ad} = V_{ae} + V_{ed} = -8 + I\cdot(3\Omega) = -6.8V$

Equivalently

$$V_{ad} = V_{ab} + V_{bc} + V_{cd} = -I\cdot(2\Omega + 5\Omega) - 4 = -6.8V$$

3-(a) $R_{13} = 4\Omega$; $R_{48} = 12\Omega$

$$\frac{1}{R_{1348}} = \frac{1}{R_{13}} + \frac{1}{R_{48}} = \frac{1}{4\Omega} + \frac{1}{12\Omega} = \frac{1}{3\Omega}$$

$\therefore R_{1348} = 3\Omega$

$$R_{ab} = R_5 + R_{1348} = 5\Omega + 3\Omega = 8\Omega$$

3-(b) The current through the 5Ω resistor is the current through $R_{ab} = R_{13458}$.

$$I_{network} = \frac{\mathcal{E}}{R_{ab}} = \frac{16V}{8\Omega} = 2A$$

$$\therefore P_{5\Omega} = I_{network}^2 \cdot (5\Omega) = 20 \text{ Watts}$$

3-(c) The potential drop between f and b, namely, R_{1348} is simply the product of $I_{network}$ times R_{1348}

$$V_{fb} = I_{network} \cdot R_{1348} = (2A) \cdot (3\Omega) = 6V$$

The current through the 8Ω resistor is the one that passes R_{48}.

$$I_{48} = \frac{V_{56}}{R_{48}} = \frac{6V}{12\Omega} = 0.5A$$

$$\therefore P_{8\Omega} = I_{48}^2 \cdot (8\Omega) = 2 \text{ Watts}$$

3-(d) The power through 4Ω resistor

$$P_{4\Omega} = I_{48}^2 \cdot (4\Omega) = 1 \text{ Watts.}$$

The current through 1Ω and 3Ω resistors is

$$I_{13} = I_{network} - I_{48} = 1.5A$$

$$P_{1\Omega} = I_{13}^2 \cdot (1\Omega) = 2.25 \text{ Watts}$$

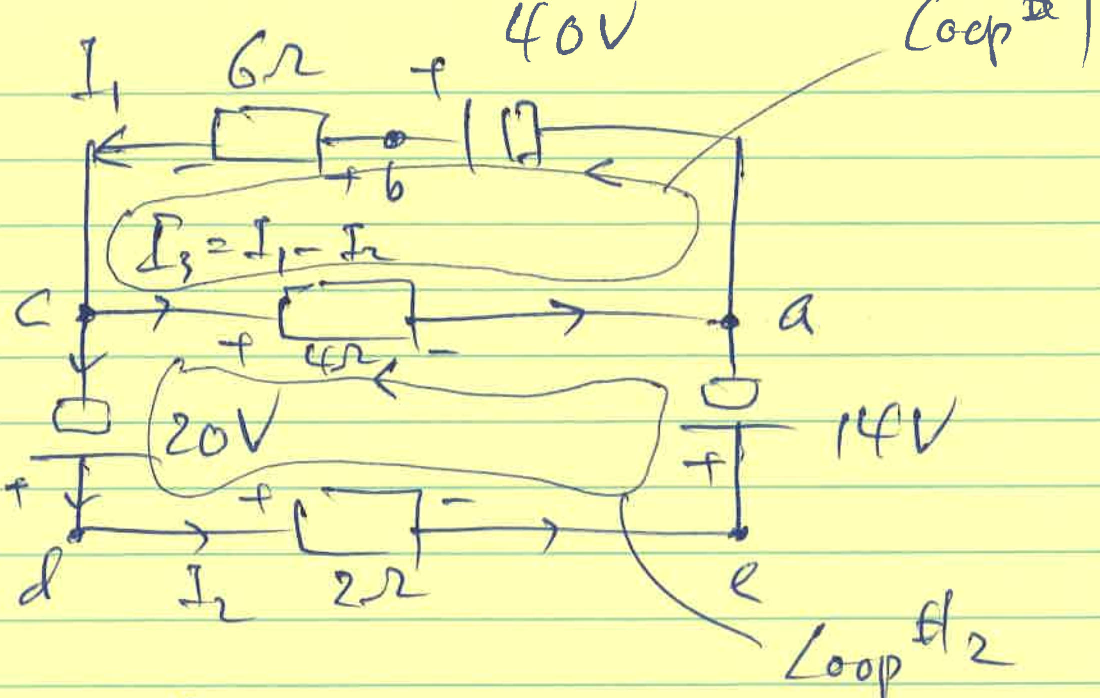
$$P_{3\Omega} = I_{13}^2 \cdot (3\Omega) = 3 P_{1\Omega} = 6.75 \text{ Watts}$$

The total power dissipated in the system is

$$P_{total} = P_{5\Omega} + P_{1\Omega} + P_{3\Omega} + P_{4\Omega} + P_{8\Omega} = 32 \text{ Watts}$$

$$P_{network} = I_{network}^2 R_{ab} = (2A)^2 \cdot (8\Omega) = 32 \text{ Watts}$$

4-(a)



Assign the currents and loops as shown

<u>Loop #1</u> :

$$\oint_{abca} \vec{E}\cdot d\vec{l} = 0 : \quad -40 + 6I_1 + 4(I_1 - I_2) = 0 \quad ①$$

$$5I_1 - 2I_2 = 20 \quad \cdots\cdots ①'$$

<u>Loop #2</u> :

$$\oint_{acdea} \vec{E}\cdot d\vec{l} = 0 : \quad -4(I_1 - I_2) - 20 + 2I_2 + 14 = 0 \quad ②$$

$$-2I_1 + 3I_2 = 3 \quad \cdots\cdots ②'$$

$3 \times ①' + 2 \times ②'$: $\quad 11 I_1 = 66 \quad \therefore I_1 = +6A.$

From ①':

$$I_2 = \frac{1}{2}(5I_1 - 20) = \frac{1}{2}(30 - 20) = 5A$$

4-(9). $V_{bd} = V_{bc} + V_{cd} = 6I_1 - 20 = 16V$

Equivaantly,

$$V_{bd} = V_{ba} + V_{ae} + V_{ed} = 40V - 14V - 2I_2$$

$$= 40V - 14V - 10V = 16V$$ #